

Parametrization of Neutrino Mixing Matrix in Tri-bimaximal Mixing Pattern

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The neutrino mixing matrix is expanded in powers of a small parameter λ in tri-bimaximal mixing pattern. We also present some applications of this parametrization, such as to the expression of the Jarlskog parameter J . Comparing with other parametrizations (such as the parametrization in bimaximal mixing pattern), this parametrization converges more quickly, but is of less symmetry.

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In recent years, the mixing of different generations of neutrinos has been established by abundant experimental data. The KamLAND [1] and SNO [2] experiments showed that the long-existed solar neutrino deficit is due to the oscillation from ν_e to a mixture of ν_μ and ν_τ with a mixing angle approximately of $\theta_{sol} \approx 34^\circ$. The K2K [3] and Super-Kamiokande [4] experiments told us that the atmospheric neutrino anomaly is caused by the ν_μ to ν_τ oscillation with almost the largest mixing angle of $\theta_{atm} \approx 45^\circ$. On the other hand, the non-observation of the disappearance of $\bar{\nu}_e$ in the CHOOZ [5] experiment indicated that the mixing angle θ_{chz} is smaller than 5° at the best fit point [6, 7].

These experiments not only confirmed the oscillations of neutrinos, but also measured the mass-squared differences of the neutrino mass eigenstates [6] (the allowed ranges at 3σ), $1.6 \times 10^{-3} \text{ eV}^2 < \Delta m_{atm}^2 = |m_3^2 - m_2^2| < 3.6 \times 10^{-3} \text{ eV}^2$, and $7.3 \times 10^{-5} \text{ eV}^2 < \Delta m_{sol}^2 = |m_2^2 - m_1^2| < 9.3 \times 10^{-5} \text{ eV}^2$, where \pm correspond to the normal and inverted mass schemes respectively.

Just like the Cabibbo-Kobayashi-Maskawa (CKM) [9] matrix for quark mixing, the neutrino mixing matrix is described by the unitary Pontecorvo-Maki-Nakawaga-Sakata (PMNS) [10] matrix V , which links the neutrino flavor eigenstates ν_e, ν_μ, ν_τ to the mass eigenstates ν_1, ν_2, ν_3 ,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu1} & V_{\mu2} & V_{\mu3} \\ V_{\tau1} & V_{\tau2} & V_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

If neutrinos are of Dirac type, the neutrino mixing matrix can be written as follows (with three mixing angles and a Dirac CP-violating phase, analogous to that of quarks)

$$V = \begin{pmatrix} c_2 c_3 & c_2 s_3 & s_2 e^{-i\delta} \\ -c_1 s_3 - s_1 s_2 c_3 e^{i\delta} & c_1 c_3 - s_1 s_2 s_3 e^{i\delta} & s_1 c_2 \\ s_1 s_3 - c_1 s_2 c_3 e^{i\delta} & -s_1 c_3 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 \end{pmatrix},$$

where $s_i = \sin \theta_i$, $c_i = \cos \theta_i$ (for $i = 1, 2, 3$), and δ is the Dirac CP-violating phase. If neutrinos are of Majorana type, it is always feasible to parametrize the neutrino mixing matrix as a product of the Dirac neutrino mixing matrix and a diagonal phase matrix with two unremovable phase angles $\text{diag}(e^{i\alpha}, e^{i\beta}, 1)$ [11], where α, β are the Majorana CP-violating phases. The Dirac CP-violating phase is associated with the neutrino oscillations, CP and T violation, and the Majorana CP-violating phases are associated with the neutrinoless double beta decay, and lepton-number-violating processes [12].

The three mixing angles θ_{atm} , θ_{chz} , and θ_{sol} are related to θ_1 , θ_2 , and θ_3 , which describe the mixing between 2nd and 3rd, 3rd and 1st, 1st and 2nd generations of neutrinos. To a good degree of accuracy, $\theta_{atm} = \theta_1$, $\theta_{chz} = \theta_2$, and $\theta_{sol} = \theta_3$.

According to the results of the global analysis of the neutrino oscillation experimental data, the elements of the modulus of the neutrino mixing matrix are summarized as follows [6]

$$|V| = \begin{pmatrix} 0.77 - 0.88 & 0.47 - 0.61 & < 0.20 \\ 0.19 - 0.52 & 0.42 - 0.73 & 0.58 - 0.82 \\ 0.20 - 0.53 & 0.44 - 0.74 & 0.56 - 0.81 \end{pmatrix}, \quad (1)$$

and the best fit points of the modulus of V are [7]

$$|V| = \begin{pmatrix} 0.84 & 0.54 & 0.08 \\ 0.44 & 0.56 & 0.72 \\ 0.32 & 0.63 & 0.69 \end{pmatrix}. \quad (2)$$

Quite different from quark mixing matrix, almost all the non-diagonal elements of the neutrino mixing matrix are large, only with the exception of V_{e3} . So it is impractical to expand the matrix in powers of one of the non-diagonal elements, like the Wolfenstein parametrization [13] of the quark mixing matrix. The quark mixing matrix is very near the unit matrix, but it is not a small modification to the unit matrix in neutrino mixing pattern. Several bases of neutrino mixing matrix are summarized as follows [14] (they all take some of the mixing

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angles as special values.)

$$\begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{6}/6 & \sqrt{6}/6 & \sqrt{6}/3 \\ \sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 \end{pmatrix}, \begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -\sqrt{2}/4 & \sqrt{6}/4 & \sqrt{2}/2 \\ \sqrt{2}/4 & -\sqrt{6}/4 & \sqrt{2}/2 \end{pmatrix},$$

$$\begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -1/2 & 1/2 & \sqrt{2}/2 \\ 1/2 & -1/2 & \sqrt{2}/2 \end{pmatrix}, \begin{pmatrix} \sqrt{6}/3 & \sqrt{3}/3 & 0 \\ -\sqrt{6}/6 & \sqrt{3}/3 & \sqrt{2}/2 \\ \sqrt{6}/6 & -\sqrt{3}/3 & \sqrt{2}/2 \end{pmatrix}.$$

Therefore we may expand the neutrino mixing matrix around these bases. The third matrix is the bimaximal mixing pattern, and the expansions around it have been discussed by Rodejohann [15], Giunti and Tanimoto [16], and us [17].

The fourth matrix is the tri-bimaximal pattern. It was first conjectured by Wolfenstein [18] long ago, and was discussed by several other authors recently [19]. It is the best approximation to the neutrino mixing matrix, and its three mixing angles are 45° , 0° and 35.3° , which agree with the experimental data perfectly. So in this paper, we will expand the neutrino mixing matrix around it.

Comparing with Eq. (2), we can make an expansion of V in powers of λ , which satisfies

$$V_{e2} = \sqrt{3}/3 - \lambda, \quad (3)$$

where λ measures the strength of the deviation of V_{e2} from the tri-bimaximal mixing pattern. Because the best fit point of V_{e2} is 0.53 [6], λ is a small parameter, which approximately equals to 0.05, and this expansion is reasonable and will converge quickly.

For the range of λ , from the analyses of the experimental data [6], we have $0.51 < V_{e2} < 0.55$ (the allowed range at 1σ). So $0.51 < \sqrt{3}/3 - \lambda < 0.55$, and we can get $0.03 < \lambda < 0.07$. Similarly, $-0.03 < \lambda < 0.1$ (the allowed range at 3σ).

Similarly, with the global analyses on the experimental data, the best fit point of $|V_{\mu 3}|^2$ is 0.52 [7]. Therefore we have $V_{\mu 3} = 0.72$, and the deviation of $V_{\mu 3}$ from $\sqrt{2}/2$ is very small, so we can set

$$V_{\mu 3} = \sqrt{2}/2 + a\lambda. \quad (4)$$

Thus $a\lambda \sim 0.013$, and $a \sim 0.3$.

Furthermore, since θ_2 is rather small (with the global analyses, $|V_{e3}| < 0.25$ (the allowed range at 3σ), and with the best fit point $|V_{e3}| = 0.08$ [7], [8]), we can set

$$V_{e3} = b\lambda e^{i\delta}. \quad (5)$$

So $b \sim 1.5$. Due to the uncertainty of the value of θ_2 , only the upper bound 0.25 is meaningful in phenomenological analyses, and the parametrization in Eq. (5) is only an assumption, however, we can adjust the value of b to satisfy the best fit point of V_{e3} , which can be determined by the long baseline experiments [20] in the future.

Altogether, there are four parameters here, a , b , λ and δ . They can describe the neutrino mixing matrix completely, both the real and the imaginary parts.

Now we will calculate all the s_i and c_i (for $i = 1, 2, 3$) to the order of λ^3 . From Eq. (5), $s_2 = b\lambda$, we have

$$c_2 = \sqrt{1 - s_2^2} = 1 - \frac{1}{2}b^2\lambda^2. \quad (6)$$

From Eq. (4), we have

$$s_1 c_2 = V_{\mu 3} = \sqrt{2}/2 + a\lambda,$$

using Eq. (6), we get

$$s_1 = \frac{\sqrt{2}}{2} + a\lambda + \frac{\sqrt{2}}{4}b^2\lambda^2 + \frac{1}{2}ab^2\lambda^3. \quad (7)$$

Similarly,

$$\begin{aligned} c_1 &= \frac{\sqrt{2}}{2} - a\lambda - (\sqrt{2}a^2 + \frac{\sqrt{2}}{4}b^2)\lambda^2 - (2a^3 + \frac{3}{2}ab^2)\lambda^3, \\ s_3 &= \frac{\sqrt{3}}{3} - \lambda + \frac{\sqrt{3}}{6}b^2\lambda^2 - \frac{1}{2}b^2\lambda^3, \\ c_3 &= \frac{\sqrt{6}}{3} + \frac{\sqrt{2}}{2}\lambda - (\frac{3\sqrt{6}}{8} + \frac{\sqrt{6}}{12}b^2)\lambda^2 \\ &\quad + (\frac{9\sqrt{2}}{16} + \frac{5\sqrt{2}}{8}b^2)\lambda^3. \end{aligned} \quad (8)$$

Thus we obtain all the trigonometric functions of the three mixing angles.

So we can get all the elements of the neutrino mixing matrix straightforwardly (to the order of λ^2),

$$\begin{aligned} V_{e1} &= \frac{\sqrt{6}}{3} + \frac{\sqrt{2}}{2}\lambda - (\frac{3\sqrt{6}}{8} + \frac{\sqrt{6}b^2}{4})\lambda^2, \\ V_{e2} &= \frac{\sqrt{3}}{3} - \lambda, \\ V_{e3} &= b\lambda e^{i\delta}, \\ V_{\mu 1} &= -\frac{\sqrt{6}}{6} + (\frac{\sqrt{2}}{2} + \frac{\sqrt{3}a}{3})\lambda - (a - \frac{\sqrt{6}a^2}{3})\lambda^2 \\ &\quad - [\frac{\sqrt{3}}{3}b\lambda + (\frac{b}{2} + \frac{\sqrt{6}ab}{3})\lambda^2]e^{i\delta}, \\ V_{\mu 2} &= \frac{\sqrt{3}}{3} + (\frac{1}{2} - \frac{\sqrt{6}a}{3})\lambda - (\frac{3\sqrt{3}}{8} + \frac{\sqrt{2}a}{2} + \frac{2\sqrt{3}a^2}{3} \\ &\quad + \frac{\sqrt{3}b^2}{4})\lambda^2 - [\frac{\sqrt{6}}{6}b\lambda - (\frac{\sqrt{2}b}{2} - \frac{\sqrt{3}ab}{3})\lambda^2]e^{i\delta}, \\ V_{\mu 3} &= \frac{\sqrt{2}}{2} + a\lambda, \\ V_{\tau 1} &= \frac{\sqrt{6}}{6} - (\frac{\sqrt{2}}{2} - \frac{\sqrt{3}a}{3})\lambda - (a - \frac{\sqrt{6}b^2}{6})\lambda^2 \\ &\quad - [\frac{\sqrt{3}}{3}b\lambda + (\frac{b}{2} - \frac{\sqrt{6}ab}{3})\lambda^2]e^{i\delta}, \\ V_{\tau 2} &= -\frac{\sqrt{3}}{3} - (\frac{1}{2} + \frac{\sqrt{6}a}{3})\lambda + (\frac{3\sqrt{3}}{8} - \frac{\sqrt{2}a}{2} - \frac{\sqrt{3}b^2}{12})\lambda^2 \\ &\quad - [\frac{\sqrt{6}}{6}b\lambda - (\frac{\sqrt{2}b}{2} + \frac{\sqrt{3}ab}{3})\lambda^2]e^{i\delta}, \\ V_{\tau 3} &= \frac{\sqrt{2}}{2} - a\lambda - (\sqrt{2}a^2 + \frac{\sqrt{2}}{2}b^2)\lambda^2. \end{aligned} \quad (9)$$

Then we can expand the neutrino mixing matrix in powers of λ (to the order of λ^2),

$$\begin{aligned}
V = & \begin{pmatrix} \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} & 0 \\ -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{pmatrix} + \lambda \begin{pmatrix} \frac{\sqrt{2}}{2} + \frac{\sqrt{3}a}{3} - \frac{\sqrt{3}}{3}be^{i\delta} & -1 & be^{i\delta} \\ -(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}a}{3}) - \frac{\sqrt{3}}{3}be^{i\delta} & -(\frac{1}{2} - \frac{\sqrt{6}a}{3}) - \frac{\sqrt{6}}{6}be^{i\delta} & a \\ -(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}a}{3}) - \frac{\sqrt{3}}{3}be^{i\delta} & -(\frac{1}{2} + \frac{\sqrt{6}a}{3}) - \frac{\sqrt{6}}{6}be^{i\delta} & -a \end{pmatrix} \\
& + \lambda^2 \begin{pmatrix} -(\frac{3\sqrt{6}}{8} + \frac{\sqrt{6}b^2}{4}) & 0 & 0 \\ -(a - \frac{\sqrt{6}a^2}{3}) - (\frac{b}{2} + \frac{\sqrt{6}ab}{3})e^{i\delta} & -(\frac{3\sqrt{3}}{8} + \frac{\sqrt{2}a}{2} + \frac{2\sqrt{3}a^2}{3} + \frac{\sqrt{3}b^2}{4}) + (\frac{\sqrt{2}b}{2} - \frac{\sqrt{3}ab}{3})e^{i\delta} & 0 \\ -(a - \frac{\sqrt{6}b^2}{6}) - (\frac{b}{2} - \frac{\sqrt{6}ab}{3})e^{i\delta} & (\frac{3\sqrt{3}}{8} - \frac{\sqrt{2}a}{2} - \frac{\sqrt{3}b^2}{12}) + (\frac{\sqrt{2}b}{2} + \frac{\sqrt{3}ab}{3})e^{i\delta} & -(\sqrt{2}a^2 + \frac{\sqrt{2}}{2}b^2) \end{pmatrix} \\
& + \dots
\end{aligned}$$

Now we will see the meaning of every order in the expansion of V .

1. The term of λ^0 is the approximation of the lowest order, where the mixing angles are of 45° , 0° and 35.3° . We call this the tri-bimaximal mixing pattern, and it is nearest to the experimental data among the bases with special mixing angles.

2. The term of λ^1 indicates the deviation of the neutrino mixing matrix from the tri-bimaximal mixing pattern. Also it shows the effect of CP violation. Because CP violation is described by the element V_{e3} [21], and in the terms of λ^0 , $V_{e3} = 0$, the degree of CP violation is of the order λ^1 in our parametrization.

3. The term of λ^2 and so on are the modifications of higher orders.

In this parametrization, several other corresponding observable quantities associated with the elements of the neutrino mixing matrix can be expressed in relatively simple forms.

1. The Jarlskog parameter J [22]. J is the rephasing-invariant measurement of the lepton CP violation. The Majorana CP-violating phases can be removed away by redefining the phases of the Dirac fields, so only δ is associated with CP violation. $J = \text{Im}(V_{e2}V_{\mu 3}V_{e3}^*V_{\mu 2}^*) = s_1s_2s_3c_1c_2^2c_3 \sin \delta$. In our parametrization, J can be expressed in a simple form (to the order of λ^2),

$$J = \frac{\sqrt{2}}{6}b\lambda \sin \delta (1 - \frac{\sqrt{3}}{2}\lambda). \quad (10)$$

Because s_1 , c_1 , s_3 , c_3 have the factors $\frac{\sqrt{2}}{2}$, $\frac{\sqrt{2}}{2}$, $\frac{\sqrt{3}}{3}$, $\frac{\sqrt{6}}{3}$, there are four factors smaller than 1 in J . So the degree of the lepton CP violation is suppressed four times, $(\frac{\sqrt{2}}{2})^2 \frac{\sqrt{3}}{3} \frac{\sqrt{6}}{3} = \frac{\sqrt{2}}{6}$. Again, J is suppressed by the factor $b\lambda \sim 0.08$ [7]. We can determine the range of J , $J \sim 0.018$. (here we take $\lambda \sim 0.05$ and $\sin \delta \sim 1$.)

2. The effective Majorana mass term $\langle m \rangle_{ee}$. In the neutrinoless double beta decay, the effective Majorana mass term is defined as follows

$$\langle m \rangle_{ee} \equiv |m_1 V_{e1}^2 e^{2i\alpha} + m_2 V_{e2}^2 e^{2i\beta} + m_3 V_{e3}^2|.$$

Using Eq. (9), we get

$$\begin{aligned}
\langle m \rangle_{ee} = & \frac{1}{3}(2m_1 e^{2i\alpha} + m_2 e^{2i\beta}) \\
& + \frac{2\sqrt{3}}{3}\lambda(m_1 e^{2i\alpha} - m_2 e^{2i\beta}) \\
& - \lambda^2[(m_1 e^{2i\alpha} - m_2 e^{2i\beta}) + b^2(m_1 e^{2i\alpha} - m_3 e^{2i\delta})].
\end{aligned}$$

We can see that the coefficients of the three terms show the influences of the three orders of λ . Only m_1 and m_2 are important to the value of $\langle m \rangle_{ee}$, and the influence of m_3 almost vanish if the masses of the three mass eigenstates are nearly degenerated, because the coefficient of it $b^2\lambda^2$ is of 10^{-3} .

3. The effective mass terms of neutrinos. The effective mass terms of neutrinos can be defined as follows (here we take electron neutrino for example.)

$$\langle m \rangle_e^2 \equiv m_1^2 |V_{e1}|^2 + m_2^2 |V_{e2}|^2 + m_3^2 |V_{e3}|^2.$$

Using Eq. (9), we get

$$\begin{aligned}
\langle m \rangle_e^2 = & \frac{1}{3}(2m_1^2 + m_2^2) - \frac{2\sqrt{3}}{3}\lambda(m_2^2 - m_1^2) \\
& + \lambda^2[(m_2^2 - m_1^2) + b^2(m_3^2 - m_1^2)]. \quad (11)
\end{aligned}$$

Again, the coefficients of the three terms show the influences of the three orders of λ . Noting that $\Delta m_{sol}^2 = |m_2^2 - m_1^2|$ and $\Delta m_{atm}^2 = |m_3^2 - m_2^2|$, we can rewrite Eq. (11) into

$$\begin{aligned}
\langle m \rangle_e^2 = & m_1^2 + [\frac{1}{3} - \frac{2\sqrt{3}}{3}\lambda + (b^2 + 1)\lambda^2](m_2^2 - m_1^2) \\
& + b^2\lambda^2(m_3^2 - m_2^2) \\
= & m_1^2 \pm [\frac{1}{3} - \frac{2\sqrt{3}}{3}\lambda + (b^2 + 1)\lambda^2]\Delta m_{sol}^2 \\
& \pm b^2\lambda^2\Delta m_{atm}^2, \quad (12)
\end{aligned}$$

where the first sign of “ \pm ” should be chosen as “ $+$ ” if we accept $m_2 > m_1$ because of Mikheyev-Smirnov-Wolfenstein (MSW) [23] matter effect on solar neutrinos. We can see from Eq. (12) that $\langle m \rangle_e^2$ is directly related with the masses and the mass-squared differences of neutrinos. So these two kinds of different observable quantities are associated together in our parametrization.

If we can separately measure Δm_{atm}^2 , Δm_{sol}^2 , and $\langle m \rangle_e^2$ to a good degree of accuracy, we can fix the value of m_1 , which will help us determine the absolute mass of neutrino ultimately.

Finally, we will give some discussion and comparison between the different methods in parametrizing the neutrino mixing matrix.

For the quark mixing, all the non-diagonal elements are small, so it is practical to expand the quark mixing matrix around the unit matrix. But the case is clearly different for the neutrino mixing. So if we still present Wolfenstein-like parametrization for the neutrino mixing matrix (as Xing did [24]), we have to use much higher orders of the non-diagonal elements. Hence it is necessary for us to find new bases for the expansion of the neutrino mixing matrix.

Among all the matrices with special mixing angles, the tri-bimaximal mixing pattern seems to be the best one. So it is natural to expand the neutrino mixing matrix around it. This is the main point of our paper. But just due to the smallness of λ ($\lambda \sim 0.05$), the expansion converges so quickly that the modulus of the matrix is rather small only to the order of λ^2 . However, in the bi-maximal case, we can expand the neutrino mixing matrix to higher orders, and can see different physical effects in different orders, because λ there is larger ($\lambda \sim 0.1$) [17]. Moreover, the expansion in tri-bimaximal mixing pattern has less symmetry than the expansion in bi-maximal mix-

ing pattern, because the mixing angles are not the same here.

Altogether, we can see that there are advantages and deficiencies at the same time in both the expansions in tri-bimaximal and bi-maximal mixing patterns, and the adoption of which of them should be determined by more and more precise experimental data.

In summary, although all sorts of parametrization of the neutrino mixing matrix are not based on any deep theoretical foundation and are equivalent mathematically, and applying any of them is arbitrary, however, it is quite likely that some particular parametrization is useful in making sense of experimental data. Furthermore, we can express some other observable quantities in a relatively simple form, and can link several different kinds of observable quantities together. This is the purpose of our parametrization.

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